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THE THREE-DIMENSIONAL STRESS–STRAIN STATE OF CROSS-PLY REINFORCED SHELLS OF REVOLUTION WHEN HEATED†

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The mechanical behaviour of cross-ply reinforced shells of revolution when they are non-axisymmetrically heated is considered in a three-dimensional formulation, and all the components of the stress–strain state are obtained in full. The method of finite elements is used for a numerical solution of the problem. The effects of anisotropy in a double-layer boroepoxide cylindrical shell under conditions of variable heating in a circumferential direction are investigated. © 2004 Elsevier Ltd. All rights reserved.

The three-dimensional form of the stress–strain state of thin non-orthogonal reinforced multilayered shells was established in the Eighties of the past century and was subject to numerous investigations by different, primarily numerical, methods, which are quite fully represented in the analytical review given in [1]. It has been shown that the presence of components of the stresses and strains, not inherent in orthotropic shells [2], distributed in complex ways and of vital importance, is determined here not so much by the thickness or large variability of the loads, but by the anisotropy of the physical-mechanical properties of the layers rigidly fastened in a single continuum of layers [3]. Even for free heating of a plane cross-ply reinforced strip a complex stressed state occurs in it, which is accompanied by the development of considerable strains of tangential shear and curvilinear warping of the cross-sections [4].

The fact that it is necessary to take into account arbitrary anisotropy in three-dimensional problems of the thermoelasticity of reinforced shells considerably complicates both the problem of obtaining numerical solutions and of analysing the stress–strain state. The first attempts at such an analysis, taking into account fairly general properties of anisotropy (non-orthotropy), were undertaken for the special case of the axisymmetric heating of shells of revolution [5, 6].

Using the Hamilton–Ostrogradskii principle and certain assumptions regarding the preliminary deformed body, the most general variational formulation and finite-element relations of three-dimensional problems of the thermoelasticity of multilayer anisotropic shells of revolution with an initial geometrically non-linear deformation were obtained in [7]. A detailed analysis of the equations obtained in [7] was carried out in [8] for a numerical solution of the linear non-axisymmetric problem of thermoelasticity, and it was pointed out that even when there is a single harmonic component of the temperature field $T = T_c^{[n]}\cos n\theta$ (or $T = T_s^{[n]}\sin n\theta$) the stress–strain state generated in an anisotropic shell is equivalent to the stress–strain state which arises when the shell is loaded with body forces with the simultaneous presence of cosine and since components.

However, a quantitative three-dimensional analysis of the effect of anisotropy on the mechanical behaviour of shells of revolution in the case of non-axisymmetric heating, with the determination of all the components of the stress–strain state in full, has not yet been carried out. The present research was undertaken with the aim of filling this gap and of continuing the cycle of investigations, carried out in [3, 7, 8] for force loading, of the stability and free vibrations of cross-ply reinforced shells. Here we also remove the inaccuracies in [5, 6] when realizing the numerical algorithms for solving the axisymmetric problem of thermoelasticity. We analyse the effects of anisotropy using the example of the stress–strain state of a non-axisymmetrically heated double-layer cross-ply reinforced boroepoxide cylindrical shell [3, 6, 8–10].

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1. THE RELATIONS OF THERMOELASTICITY FOR A MULTILAYER CROSS-PLY REINFORCED SHELL

In cylindrical coordinates r, θ, z we consider a shell of revolution, bounded by a surface S and having a volume V [7, 8]. The shell is free from surface loads and external body forces. Assuming the increment of temperature with respect to the initial state T_0 is known and constant with time, we will assume the deformation of the shell to be isothermal, in which case the stresses do not depend on the prehistory of the loading [11].

It is most convenient to analyse the stress–strain state in a system of coordinates α_1, α_2, α_3, connected with the internal surface of the shell, where α_1 and α_2 are the meridional and circumferential coordinates, while the α_3 axis is directed along the outer normal to α_1 and α_2. The shell consists of rigidly connected unidirectional reinforced layers, arranged in such a way that the directions of the reinforcement make angles $(-1)^{k+1}\gamma$ with the tangent to the meridian (k is the number of the layer measured from the internal surface). In axes connected with the direction of the reinforcement, each layer is assumed to be macroscopically transversely isotropic. The reduced mechanical characteristics of a layer are determined from well-known averaging formulae [12], and the coefficients of thermal expansion are calculated with the same accuracy using well-known relations [5]. Taking the form of the reinforcement into account, such a layer, in the connected system of coordinates α_1, α_2, α_3, possesses the properties of cylindrical anisotropy with a single plane of elastic symmetry, tangential to the surface, equivalent to the coordinate surface. The material of the layers corresponds to the physical Duhamel–Neumann relations

$$\sigma^{(k)} = C^{(k)}\varepsilon^{(k)} - T\beta^{(k)} \tag{1.1}$$

represented in matrix form. Here $\sigma = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}]^T$ is the stress vector, $C_{ij} = C_{ij}(i, j = 1, 2, \dots, 6)$ is the stiffness matrix of the generalized Hooke's law (it contains 13 different non-zero components), $\varepsilon = [\varepsilon_{11}, \dots, \varepsilon_{12}]^T$ is the strain vector, T is the increment of the temperature with respect to the initial state T_0 (a scalar field), and β is the temperature stress vector for constrained deformations, equal to $\beta = C\alpha$, where $\alpha = [\alpha_{11}, \alpha_{22}, \alpha_{33}, 0, 0, \alpha_{12}]^T$ is the vector of thermal expansions and shears.

The following equations hold for the non-zero elasticity constants and components of the vector α of the kth and $(k + 1)$th layers

$$\begin{aligned} &C_{ii}^{(k)} = C_{ii}^{(k+1)}, \quad C_{jq}^{(k)} = C_{jq}^{(k+1)}, \quad C_{j6}^{(k+1)} = -C_{j6}^{(k+1)}, \quad \alpha_{jj}^{(k)} = \alpha_{jj}^{(k+1)} \\ &i = 1, 2, \dots, 6; \quad j, q = 1, 2, 3 \\ &C_{45}^{(k)} = -C_{45}^{(k+1)}, \quad \alpha_{12}^{(k)} = -\alpha_{12}^{(k+1)} \end{aligned} \tag{1.2}$$

The following conditions of continuity hold on the boundary of the layers

$$\begin{aligned} &u_i^{(k)} = u_i^{(k+1)}, \quad \sigma_{i3}^{(k)} = \sigma_{i3}^{(k+1)}, \quad \varepsilon_{pl}^{(k)} = \varepsilon_{pl}^{(k+1)} \\ &i = 1, 2, 3; \quad p, l = 1, 2 \end{aligned} \tag{1.3}$$

If we change to a cylindrical system of coordinates, the layers considered will possess anisotropy of general form, and instead of (1.1) for

$$\sigma = [\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{\theta z}, \theta_{rz}, \sigma_{r\theta}]^T, \quad \varepsilon = [\varepsilon_{rr}, \dots, \varepsilon_{r\theta}]^T$$

we will have

$$\sigma^{(k)} = B^{(k)}\varepsilon^{(k)} - T\beta^{(k)}, \quad \beta^{(k)} = B^{(k)}\alpha^{(k)} \tag{1.4}$$

The components of the vector ε are determined from well-known relations [11], the stiffness matrix B has 21 non-zero components, and $\alpha = [\alpha_{rr}, \dots, \alpha_{r\theta}]^T$.

The characteristics of a cylindrical shell, as a special case, are given by

$$\begin{aligned} &B_{ii}^{(k)} = B_{ii}^{(k+1)}, \quad B_{jq}^{(k)} = B_{jq}^{(k+1)}, \quad B_{j4}^{(k)} = -B_{j4}^{(k+1)}, \quad B_{56}^{(k)} = -B_{56}^{(k+1)} \\ &i = 1, 2, \dots, 6; \quad j, q = 1, 2, 3 \\ &\alpha_{rr}^{(k)} = \alpha_{rr}^{(k+1)}, \quad \alpha_{\theta z}^{(k)} = -\alpha_{\theta z}^{(k+1)} (rr \rightleftarrows \theta\theta \rightleftarrows zz) \end{aligned} \tag{1.5}$$

The remaining components of the stiffness matrix B and the vector of the thermal expansions and shears α of the physical Duhamel–Neumann relations (1.4) are zero.

The differences in the signs of the components $C_{j6}(j = 1, 2, 3)$, C_{45} and α_{12} (1.2) of adjacent layer gives rise to effects of non-uniformity of the stress and strain fields. Effects connected with the anisotropy for shells of different thickness manifest themselves most strongly when there is a small number of layers, and less so when the number of layers is increased [9].

2. THE FINITE-ELEMENT SOLUTION OF THE PROBLEM OF THE NON-AXISYMMETRIC DEFORMATION OF A SHELL OF REVOLUTION WHEN HEATED

A numerical solution of the three-dimensional problem of the theory of elasticity for multilayer anisotropic shells of revolution for non-axisymmetric force deformation was obtained for the first time by Grigolyuk and Nosatenko [3]. Here we will use the special case of the linear equations of static equilibrium [7, 8], which follow from the variational principle of the minimum of the total energy for the isothermal state of an arbitrarily heated elastic body (unloaded by surface and body forces). The stress–strain state of cross-ply reinforced shells is characterized by the displacements $u_\alpha(r, \theta, z)$ ($\alpha = r, \theta, z$), the vectors of the linear deformations $\varepsilon(r, \theta, z)$ and the stresses $\sigma(r, \theta, z)$, comprised of the components of the corresponding tensors [11] and connected by the Duhamel–Neumann relations (1.4).

The components $u_\alpha(\alpha = r, \theta, z)$, ε and σ, and also the increment of the temperature T, are represented in the form of Fourier series in the circumferential coordinate

$$u_\alpha = \sum_{n=0}^{\infty} u_\alpha^{[n]} \quad (\alpha = r, \theta, z), \quad \varepsilon = \sum_{n=0}^{\infty} \varepsilon^{[n]} \quad (\varepsilon \rightleftarrows \sigma) \tag{2.1}$$

$$T = \sum_{n=0}^{\infty} (T_c^{[n]} \cos n\theta + T_s^{[n]} \sin n\theta) \tag{2.2}$$

In relations (2.1)

$$\begin{aligned} u_r^{[n]} &= v_r^{[n]}(r, z)\cos n\theta + w_r^{[n]}(r, z)\sin n\theta \quad (r \rightleftarrows z) \\ u_\theta^{[n]} &= v_\theta^{[n]}(r, z)\sin n\theta + w_\theta^{[n]}(r, z)\cos n\theta \end{aligned} \tag{2.3}$$

The components of the vectors $\sigma^{[n]}$ and $\varepsilon^{[n]}$ have the form

$$\begin{aligned} &\sigma_{rr}^{[n]} = \tau_{rr}^{[n]}(r, z)\cos n\theta + t_{rr}^{[n]}(r, z)\sin n\theta \quad (rr \rightleftarrows \theta\theta \rightleftarrows zz \rightleftarrows rz) \\ &\sigma_{\theta z}^{[n]} = \tau_{\theta z}^{[n]}(r, z)\sin n\theta + t_{\theta z}^{[n]}(r, z)\cos n\theta \quad (\theta z \rightleftarrows r\theta) \\ &(\sigma \to \varepsilon \Rightarrow \tau \to e, t \to g) \end{aligned} \tag{2.4}$$

In the case of an orthotropic shell, in which, for each layer, the directions of orthotropy coincide with the curvilinear coordinates α_1, α_2 α_3, from expressions (1.4) we obtain the equations

$$\tau^{[n]} = Be^{[n]} - T_c^{[n]}\beta, \quad t^{[n]} = Bg^{[n]} - T_s^{[n]}\beta \tag{2.5}$$

If anisotropy of a more general form than curvilinear orthotropy is taken into account, equations of the type (2.5) are impossible, since the components τ and t are related by Eqs (1.4) both with he components e and g.

The problem is solved by the finite-element method with linear local approximations of the expansion coefficients (2.3) $v_\alpha^e(r, z)$ and $w_\alpha^e(r, z)$ ($\alpha = r, \theta, z$) in triangles Ω_e (here and below the subscript e denotes membership of the eth finite element, and we will omit the superscript n, implying that all the relations are referred to the harmonic n of expansions (2.1)–(2.4)). To calculate the vector of the thermal stresses we will assume that the temperature field (the coefficients of expansion (2.2)) are specified by the values at the nodes of a discrete model; in the region $r, z \in \Omega_e$ we take the linear approximation of the temperature field. As was shown in [5], this ensures that the calculated models for the force and thermal loading agree.

The problem of finding the stress–strain state for the case of non-axisymmetric heating reduces to the linear algebraic relations [8]

$$K_L U = Q_T \tag{2.6}$$

with subsequent calculation of the strains ε and stresses σ at the nodes of the discrete model and their conversion to the coordinates α_1, α_2, α_3.

In relations (2.6)

$$U = [\upsilon_r^1, w_\theta^1, \upsilon_z^1, w_r^1, \upsilon_\theta^1, w_z^1, \upsilon_r^2, \ldots, w_z^{N_p}]^T \quad (\dim U = 6N_p)$$

$$[K_{L_{ij}}] = \left[\frac{\partial^2}{\partial U_i \partial U_j}\sum_{e=1}^{N_e}\frac{1}{2}\iint_{\Omega_e}\{(\varepsilon_c^e)^T B^e \varepsilon_c^e + (\varepsilon_s^e)^T B^e \varepsilon_s^e\} r dr dz\right]$$

$$\{Q_{T_i}\} = \left\{\frac{\partial}{\partial U_i}\sum_{e=1}^{N_e}\iint_{\Omega_e}[T_c^e(\beta^e)^T\varepsilon_c^e + T_s^e(\beta^e)^T\varepsilon_s^e] r dr dz\right\}$$

$$\varepsilon_c^e = [e_{rr}^e, e_{\theta\theta}^e, e_{zz}^e, g_{\theta z}^e, e_{rz}^e, g_{r\theta}^e]^T \quad (c \to s \Rightarrow e \rightleftarrows g)$$

where U is the global vector of the generalized displacements, N_p is the total number of nodes of the discrete model, $\lfloor K_{L_{ij}} \rfloor$ is a square symmetrical positive-definite matrix of the stiffness, N_e is the total number of finite elements and $\{Q_{T_i}\}$ is the vector of the thermal loads.

Note that when $n = 0$, the solution of the axisymmetric linear problem of thermoelasticity follows from relations (2.1)–(2.6).

When analysing the heating of a cross-ply reinforced shell, considered as orthotropic, by virtue of relations (2.5) the problems of determining the coefficients $\upsilon_\alpha(r, z)$ and $w_\alpha(r, z)$ ($\alpha = r, \theta, z$) in expansion (2.1), (2.3) are independent. This, of course, enables us to halve the dimension of the resolvents (2.6), but it eliminates the taking into account of the mutual influence of the different harmonic components of the stress–strain state, characteristic of the anisotropy itself [3].

When using an expansion of the temperature field only in cosines ($T_s^{[n]} = 0$) instead of (2.2), the calculation using the orthotropy model leads to the occurrence of only the displacements $\upsilon_\alpha(r, z)$ and the related strains $e_{\alpha\beta}(r, z)$ and stresses $\tau_{\alpha\beta}(r, z)$ (α, $\beta = r, \theta, z$), which will be called the "fundamental" components of the stress–strain state. The contribution of the anisotropy, in addition to the change in the values of the "fundamental" components of the stress–strain state, is then characterized by the values of the "additional" displacement $w_\alpha(r, z)$, the strains $g_{\alpha\beta}(r, z)$ and the stresses $t_{\alpha\beta}(r, z)$ (α, $\beta = r, \theta, z$).

The relations for the components of the vector Q_T and of the matrix K_L of problem (2.6) were obtained in analytical form, which ensures the highest accuracy of the numerical solution within the framework of the formulation and approximation assumed (the use of numerical integration in problems of linear statics leads to a several-fold degradation of the convergence for individual cases).

3. ANALYSIS OF THE EFFECT OF ANISOTROPY ON THE STRESS–STRAIN STATE OF A CROSS-PLY REINFORCED SHELL WHEN HEATED

We will investigate the properties of the stress–strain using the example of a cylindrical shell consisting of two rigidly clamped layers of the same thickness $H/2$, unindirectionally reinforced at angles $\gamma^{(1)} = 30°$ and $\gamma^{(2)} = -30°$. Each layer of the shell is made of boroplastic. The mechanical characteristics of the structural components correspond completely to those assumed in [3, 6, 8, 9]: $E_f = 4.2 \times 10^5$ MPa, $\nu_f = 0.21$, $E_m = 3.5 \times 10^3$ MPa, $\nu_m = 0.33$ and a volume reinforcement coefficient $\psi = 0.5$. The coefficients of linear expansion of the binding $\alpha_m = 1.14 \times 10^{-5}$ °C^{-1} and of the fibres $\alpha_f = 8.25 \times 10^{-6}$ °C^{-1} are taken from [13, 14]. The geometrical dimensions of the shell are the same as in [3, 6, 8, 9].

In order to estimate the effect of anisotropy on the stress–strain state of cross-ply reinforced shells, as was done previously in [8], we specially considered the case of a macroscopically uniform orthotropic shell. To do this the components of the stiffness matrix and of the vector of the thermal expansions and shears (1.2), (1.5) of the physical Duhamel–Neumann relations, with adjacent layers having a different sign, were assumed to be equal to zero (as follows from the principle of averaging the values of the physical and mechanical characteristics over the volume)

Table 1

n	$v_1^o \times 10^4$	$v_1^a \times 10^4$	$w_1^a \times 10^5$	$v_2^o \times 10^4$	$v_2^a \times 10^4$	$w_2^a \times 10^4$	$v_3^o \times 10^3$	$v_3^a \times 10^3$	$w_3^a \times 10^4$
0	61.2	72.3	–	–	–	163	263	313	–
1	61.8	72.7	248	76.4	47.9	158	254	306	57.2
2	63.9	73.9	457	155	100	144	229	284	101
4	76.7	81	644	312	226	94	146	203	129
5	86.8	87.7	599	376	290	66.4	101	151	121
7	102	101	338	433	372	26.1	35.4	60.2	102
8	103	104	200	425	378	16	17.6	31.4	96.5
11	81.7	85.9	48.4	329	304	5.99	4.75	6.8	74.7
15	54.4	56.6	43.3	209	189	3.29	2.39	3.42	39.7

Table 2

n	$u_3^o \times 10^3$	$u_3^a(\theta^*) \times 10^3$	θ^*
1	128	149	–1.96
2	118	140	–1.85
5	58.2	82.6	–1.56
6	38.9	58.8	–1.68
8	14.9	25.1	–2.70
11	4.44	9.92	–4.38
15	1.66	4.62	–3.58

$$C_{j6}^{(k)} = C_{45}^{(k)} = 0, \quad B_{j4}^{(k)} = B_{56}^{(k)} = 0, \quad j = 1, 2, 3$$
$$\alpha_{12}^{(k)} = 0, \quad \alpha_{\theta z}^{(k)} = 0, \quad k = 1, 2$$

The calculation was carried out for the case when the shell was rigidly fastened at the ends $z = 0$ and $z = L$ and for a temperature field which changed only in a circumferential direction, $T = 100\cos n\theta$ °C over a range of variability of the load $0 \le n \le 15$.

Such an elastic system is symmetrical in its geometry and fixing about the central section of the shell $z = L/2$ to the plane normal to its axis. In the tables and the figures, which represent the results of the calculation, the linear dimensions are in millimeters, the angular quantities are in degrees and the stresses are in megapascals. The continuous curve in the figures corresponds to the solution for an anisotropic shell, and the dashed curve is for an orthotropic shell.

The maximum absolute values of the fundamental displacements v_α and the additional displacements w_α ($\alpha = 1, 2, 3$) are shown in Table 1 (here and below the superscript a corresponds to an anisotropic shell while the superscript o corresponds to an orthotropic shell). As might have been expected, in view of the presence of non-zero constrained deformations of the tangential shear $T\alpha_{12}$, the anisotropy mainly affects the values of the fundamental circumferential displacements υ_2 and deflections υ_3. The effect of the anisotropy manifests itself more strongly for $n = 8$ (in this case $\upsilon_3^a/\upsilon_3^o = 1.78$) and more weakly for $n = 0(\upsilon_3^a/\upsilon_3^o = 1.19)$. Essentially, as the load variability parameter increases ($n > 10$) the additional displacements w_3 already exceed the fundamental bucklings υ_3.

For axisymmetric loading ($n = 0$) the stress–strain state was calculated both using the method presented here and using the more accurate AAMS algorithm [5], taking the geometrical non-linearity into account. The differences between the linear and non-linear solutions amounted to less than 5% (which is fully explainable by the smallness of the coefficient of linear expansion of the extremely rigid filler).

The maximum buckling u_3^* for an orthotropic body, by virtue of expansion (2.3), corresponds to $\theta = 0$, which is not the case for an anisotropic shell, in each specific section of which the maximum buckling u_3^* occurs for a quite definite angular coordinate

$$\theta^* = \frac{1}{n}\operatorname{arctg}\frac{w_3}{\upsilon_3}$$

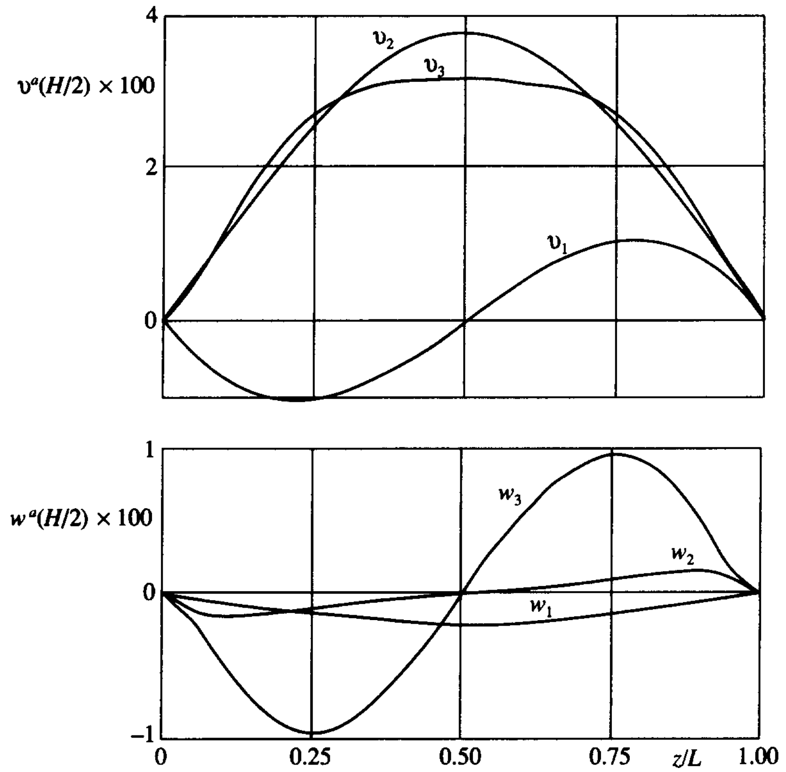

Fig. 1

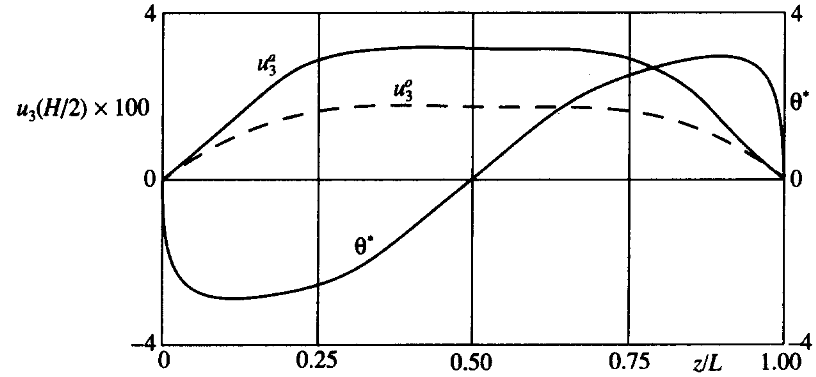

Fig. 2

In Table 2, for the section $z = 0.2\,L$, in which the additional components of the displacements reach considerable values, we compare the maximum values of the bucklings in an orthotropic shell u_3^o and an isotropic shell $u_3^a(\theta^*)$. For $n > 10$, when the additional displacements w_3 are commensurable in value with the fundamental bucklings υ_3, neglect of the anisotropy introduces an error into the calculated values of the bucklings of up to 100% or more.

We carried out a further analysis for the case $n = 8$.

In Fig. 1 we show the distributions of the fundamental and additional displacements of the middle surface of an anisotropic shell along the length (for an orthotropic shell the displacements have the same as υ_α^a). For υ_2, υ_3, and w_1 the central section is the plane of symmetry. The components w_2, w_3 and υ_1 are antisymmetric about the same section.

The distributions of the maximum bucklings of the middle surface of an orthotropic and an anisotropic shell along the length are shown in Fig. 2. Here the curve of θ^* represents the twisting of the anisotropic

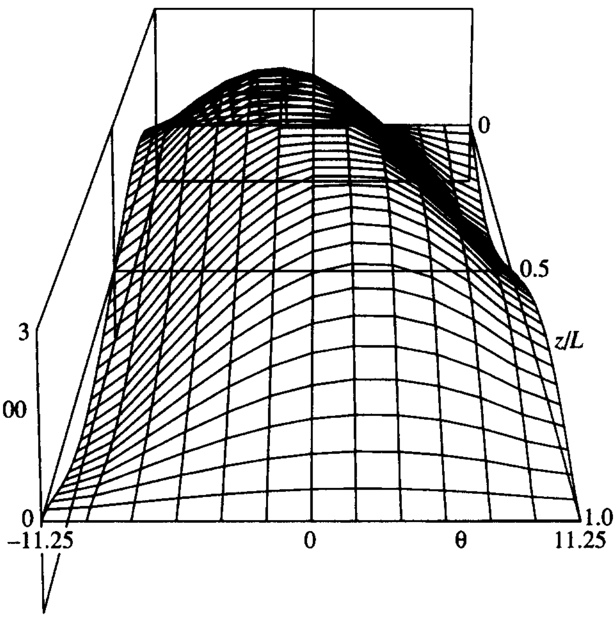

Fig. 3

shell, when the line of maximum bucklings becomes a curve which does not coincide with the generatrix. This effect is clearly shown in Fig. 3, where, we show, relative to the plane of development of the fragment of the middle surface of the anisotropic shell in the region D: $\{0 \leq z/L \leq 1, -\pi/(2n) \leq \theta \leq \pi/(2n)\}$, the form of its bucklings, which occur during thermal deformation.

Considering the deformed cross-ply reinforced cylinder as a fine three-dimensional figure, we must point out the change in the properties of symmetry [15], which arises when anisotropy is taken into account. Whereas in the orthotropic formulation, the cylinder in the deformed state, as in the initial state, has a plane of symmetry, perpendicular to the cylinder axis and dividing it into two equal parts (the point of intersection of the cylinder axis in this plane $\{z = L/2, r = 0\}$ is a singular point), when the anisotropy is taken into account there is only a second-order axis of symmetry, coinciding with the radial coordinate line drawn through the same singular point.

The distributions over the thickness of all the components of the fundamental and additional stresses in anisotropic and orthotropic shells in sections where they take extremum values, are shown in Fig. 4 for shear stresses, and in Fig. 5 for transverse stresses. Narrow zones, directly next to the clamping, where the elastic solution has local singularities, are excluded from consideration. In the sections considered the additional shear stresses $t_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) are practically self-balancing over the thickness:

$$\int_0^H t_{\alpha\beta} d\alpha_3 \approx 0$$

It follows from the results obtained that the stresses $t_{\alpha\beta}$ and $\tau_{\alpha\beta}$ are close in absolute value and take maximum values in different sections along the meridional coordinate. This confirms that there are equal stresses over the whole volume of the cross-ply reinforced shell.

For the "fundamental" components of the stress–strain state of orthotropic and anisotropic shells, the overall properties of symmetry about the central section of the shell $z = L/2$ are characteristic.

$$\begin{aligned} &v_\alpha\left(\frac{L}{2}+z\right) = v_\alpha\left(\frac{L}{2}-z\right), \quad \alpha = 2, 3; \quad \tau_{11}\left(\frac{L}{2}+z\right) = \tau_{11}\left(\frac{L}{2}-z\right) \\ &(11 \rightleftarrows 22 \rightleftarrows 33 \rightleftarrows 23) \\ &v_1\left(\frac{L}{2}+z\right) = -v_1\left(\frac{L}{2}-z\right), \quad \tau_{13}\left(\frac{L}{2}+z\right) = -\tau_{13}\left(\frac{L}{2}-z\right) \quad (13 \rightleftarrows 12) \end{aligned} \tag{3.1}$$

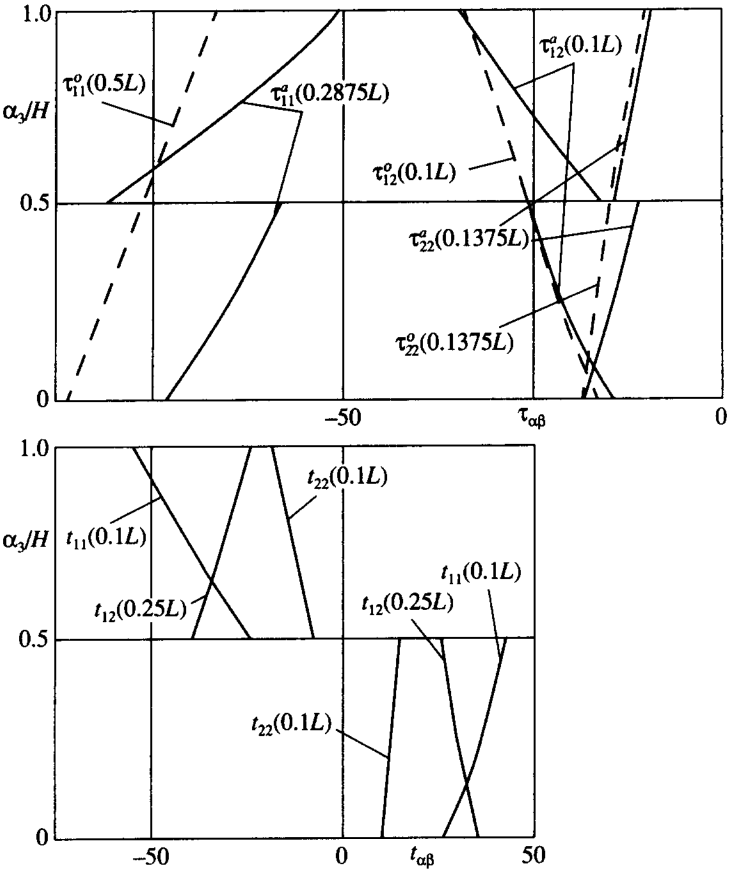

Fig. 4

For the "additional" components, due to the anisotropy, they are opposed

$$w_\alpha\left(\frac{L}{2}+z\right) = -w_\alpha\left(\frac{L}{2}-z\right), \quad \alpha = 2, 3; \quad t_{11}\left(\frac{L}{2}+z\right) = -t_{11}\left(\frac{L}{2}-z\right)$$

$$(11 \rightleftarrows 22 \rightleftarrows 33 \rightleftarrows 23)$$

$$w_1\left(\frac{L}{2}+z\right) = w_1\left(\frac{L}{2}-z\right), \quad t_{13}\left(\frac{L}{2}+z\right) = t_{13}\left(\frac{L}{2}-z\right) \quad (13 \rightleftarrows 12) \tag{3.2}$$

Hence, the general properties of symmetry (3.1) and (3.2), established for the first time in [3] and discussed in detail in [8] for three-dimensional problems of force and kinematic non-axisymmetric deformation, also occur during heating. This enables us to use the geometrical and elastic symmetry for a numerical solution of three-dimensional problems of the mechanics of multilayer cross-ply reinforced shells in the case of complex thermal loading (when finding a solution in displacements in the plane of symmetry $\upsilon_1 = w_2 = w_3 = 0$).

4. CONCLUSIONS

Summarizing the main results obtained in this paper and in [3, 7, 8], we point out the following general rules.

1. Neglect of the anisotropy and of the three-dimensional nature of the stress–strain state leads to a considerable increase in the stiffness of multilayer reinforced shells.

2. The more complex the phenomenon investigated, the more important the contribution of the anisotropy in estimating its fundamental characteristics (in the vibrations of a previously deformed shell this is a reduction and readjustment of the whole spectrum of elastic oscillations, while in problems of

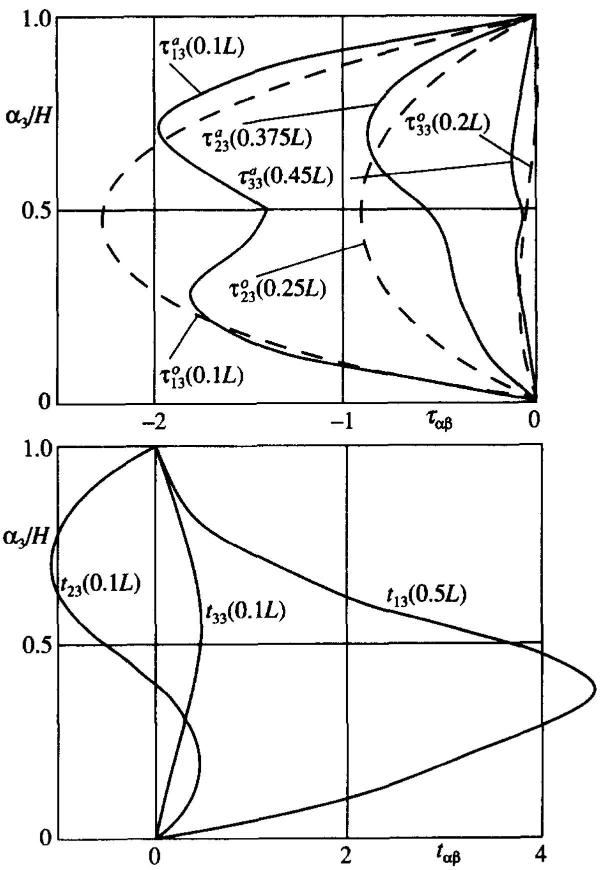

Fig. 5

statics this will be additional transverse–shear stresses, which will affect the strength of composite materials in a definite way).

3. In all cases, non-axisymmetric deformation of anisotropic shells is accompanied by twisting of the shells, when the line of maximum bucklings becomes a curve which does not coincide with the generatrix.

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